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Direct Model Reference Takagi–Sugeno Fuzzy Control of SISO Nonlinear Systems Design by Membership Function

Foad Hosseini[1], Meshkat Sadat Hosseini[2]

[1] Semnan University, Semnan, Iran, Foad.hoseyni@gmail.com
[2] Shahed University, Tehran, Iran, meshkat.hoseini89@gmail.com

*Correspondence:
Foad Hosseini, Semnan University, Semnan, Iran,
Foad.hoseyni@gmail.com

Abstract

What is discussed in this article is to find a way for membership functions optimally. In most scholars, these functions are constant and have a limited number. Therefore, in some cases, this limitation reduces control performance improvement. One of the best solutions is finding these functions in a differential form. This article used the Takagi-Sugeno function as a fuzzy detector to identify and control a nonlinear SISO system by direct adaptive reference model control. Using this method with Lyapunov for the stability of the control system makes output fuzzy linguistic variables optimally. Then simultaneously using these values, membership functions can be defined in differential form. Therefore, there is no other limitation in the variance and midpoint.

Keyword: Fuzzy Control, Model Reference Adaptive Control, Takagi-Sugeno (T-S) Fuzzy Model, Membership Function, Fuzzy Inference Engine

1. Introduction

Finding the best way to control nonlinear systems is the highest important issue. Various methods have been presented for controlling such systems so far. In (Unbehauen, H. D. (Ed.), 2009), one of the essential methods is written, "How to control a nonlinear system using sliding mode method." This method has many disadvantages, including a chattering phenomenon at high frequencies. The other nonlinear control method, "Backstepping," is perused in (Madani, T. and Benallegue, A., 2006). This method needs the system's dynamic to be tangible.

Otherwise, when the system's parameters change over time, it does not work, and adaptive control is the best choice. In (Åström, K.J. and Wittenmark, B., 2013), an adaptive controller was used to solve variable parameters and get the best result for the following reference signal. However, it only works for systems with sudden changes in them. In this case, the use of a robust method is suggested. It is difficult to adapt to new conditions due to sudden changes in parameters such as altitude, airplane change, and speed changes (Zimmermann, H.J., 1996; Malki, H.A., Misir, D., Feigenspan, D. and Chen, G., 1997; Huang, A.C. and Chen, Y.C., 2004). Another control methods are the fuzzy method, which brought about a significant transformation in control science. By this means, the use of actual numbers is not applicable. It can use fuzzy concepts to control the system, however. In (Bridges, M.M., Dawson, D.M. and Abdallah, C.T., 1995), how to control using. In most articles, the combination of the described control methods is employed as necessary to achieve the best process control of a system. One of the most effective systems is due to the unpaired and nonlinear features of the flexible joint robot system, which has been designed to control it. In (Green, A. and Sasiadek, J.Z., 2005), the use of the Fuzzy PID control method to control the uncertainty of the system is discussed. In (Cho, Y.W., Park, C.W. and Park, M., 2002), this system is controlled by the adaptive manipulation method. As well as in (Khanesar, M.A., Kaynak, O. and Teshnehlab, M., 2011), this system is controlled using a synchronous and adaptive combination method, and (11) examined comparative fuzzy method control.

Therefore, using the fuzzy method has higher flexibility and, consequently. The only content that should be discussed in the papers is finding an optimal differential equation for membership functions. In many papers that road before, membership functions considered in the fuzzy inference engine have constant category center and variance. This paper considered Takagi-Sugeno as the system's dynamic model. Concerning the error value, we used the Lyapunov method. Eventually, by Lyapunov method obtained optimal amounts of data center and variance.

2. Design of Direct Model Reference Fuzzy Controller

a. Fuzzy Takagi-Sugeno Modeling

It assumed that the primary dynamics of the system are expressed in terms of the Takagi-Sugeno model. For this purpose, the system is defined as a set of if-then rules. It seeks to express a system with nonlinear dynamics as a combination of several linear models. It is considered system behaves continuously in time. Therefore express, the law of a continuous system in time with the Takagi-Sugenu model is:

$$\mathrm{Re}: IF\ x_1\ is\ c_1^i\ and\ \dots and\ x_n\ is\ c_n^i$$
$$THEN\ \dot{x} = A_i x + B_i u. \quad i = 1.2.\dots.m$$

Where $x^T(t) = [x_1.x_2.\dots.x_n]$ and m is the number of rules of the T-S system. Also, A_i and B_i $(i = 1.\dots.m)$ are the state matrixes of the system whose constituent elements are the number of fuzzy rules. This hypothesis is available for most systems (Liu, Y.J., Tong, S. and Chen, C.P., 2013). We want to show the hypothesized system in the form of a fuzzy:

$$\dot{x} = \frac{\sum_{i=1}^{m} w_i\big(x(t)\big)\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{m} w_i\big(x(t)\big)} \tag{1}$$

Where $w_i\big(x(t)\big) = \prod_{j=1}^{n} M_j^i(x(t))$ and $M_j^i(x(t))$ are the grades of membership function of $x_j(t)$ inC_j^i.

b. Designing a Fuzzy Controller Reference Model

It is assumed to follow the reference signal system. Consequently, fuzzy rules and control signals act in such a way as to obey the reference signal. Suppose the relationship of the reference signal is as follows:

$$\dot{x}_m = A_m x_m + B_r r \tag{2}$$

For any fuzzy rules, the control signal takes a certain amount. As we have:

$$A_m = A_i - B_i K^*{}_i \ \ . \ B_r = B_i l^*{}_i \tag{3}$$

The control signal is obtained in the following form:

$$u = \frac{\sum_{i=1}^{m} w_i |l_i|^{-1} [-K_i x + l_i r]}{\sum_{i=1}^{m} w_i \, |l_i|^{-1}} \tag{4}$$

The most critical issue is stability. If the system were not stable, control would not work. Thus measuring error and its derivative would be significant. That obtained as follows:

$$\dot{e}_m = A_m e_m + \frac{\sum_{i=1}^{m} w_i B_r \widetilde{K}_i l_i^{*-1} x}{\sum_{i=1}^{m} w_i} + \\ + \frac{\sum_{i=1}^{m} w_i B_r K_i (l_i^{*-1} - l_i^{-1}) x}{\sum_{i=1}^{m} w_i} + \\ + \frac{\sum_{i=1}^{m} w_i B_r (l_i^{*-1} - l_i^{-1}) u}{\sum_{i=1}^{m} w_i} \tag{5}$$

In this regard, it is assumed:

$$\widetilde{K}_i = K_i^* - K_i$$

What important is the stability of the system? Lyapunov Theorem: for any system, if it is possible to find a positive function whose derivative is negative, the Lyapunov stability can achieve for the system.

In this section, the Lyapunov function considers error value and adaptation lows (6).

$$V = e_m^T P e_m + \frac{\sum_{i=1}^{m} \widetilde{K}_i^T |l_i^*|^{-1} \widetilde{K}_i}{\gamma_1} + \frac{\sum_{i=1}^{m} \tilde{l}_i^T |l_i^*|^{-1} \tilde{l}_i}{\gamma_2} \tag{6}$$

In this regard, as in the past: $\tilde{l}_i = l_i^* - l_i$

Now, to prove the stability, we derive from the Lyapunov function:

$$\dot{V} = e_m^T P A_m e_m + e_m^T A_m^T P e_m \\ + 2 e_m^T P \left\{ \frac{\sum_{i=1}^{m} w_i B_r \widetilde{K}_i l_i^{*-1} x}{\sum_{i=1}^{m} w_i} + \frac{\sum_{i=1}^{m} w_i B_r (l_i^{*-1} - l_i^{-1}) u}{\sum_{i=1}^{m} w_i} \right. \\ \left. + \frac{\sum_{i=1}^{m} w_i B_r K_i (l_i^{*-1} - l_i^{-1}) x}{\sum_{i=1}^{m} w_i} \right\} + \frac{2}{\gamma_1} \sum_{i=1}^{m} \widetilde{K}_i^T |l_i^{*-1}| \dot{\widetilde{K}}_i + \frac{2}{\gamma_2} \sum_{i=1}^{m} \tilde{l}_i^T |l_i^{*-1}| \dot{\tilde{l}}_i \tag{7}$$

Adaptation laws are as follows:

$$\dot{K}_i = -\dot{\widetilde{K}}_i = \gamma_1 sign(l_i^*) \frac{w_i B_r^t P e_m x^T}{\sum_{i=1}^{m} w_i} \ . \gamma_1 > 0 \tag{8}$$

$$\dot{l}_i = -\dot{\tilde{l}}_i = \gamma_2 sign(l_i^*) \frac{w_i B_r^t P e_m (x + K_i x)}{l_i \sum_{i=1}^{m} w_i} \ . \ \gamma_2 > 0 \tag{9}$$

By use of these adaptation laws:

$$\dot{V} = -e_m^T Q e_m \tag{10}$$

3. Obtain the Differential Equation for the Center of the Class and the Variance

In many articles, the modulus and variances of the membership functions are assumed to be constant. Constant amounts can decrease control optimally. For this reason, this article's scrutiny is optimizing these values. They have taken membership function in optimal amount. Writing the differential equation for the membership function is essential for the best response. It makes increase the flexibility of the control system. If you need a membership function with a different center or variance, you change the number of this equation. The other most crucial matter is that these equations must be optimized, and there must be consistency with the optimal control of the system.

$$\dot{e} = A_m e + \sum h_i^* B_r l_i^{*-1} k_i^* x + \sum h_i^* B_r l_i^* u + \sum h_i B_r l_i^{-1}(-k_i x + l_i r) \quad (11)$$

$$= A_m e + \sum h_i^* B_r l_i^* x + \sum h_i^* B_r l_i^{*-1} u + \sum h_i^* B_r l_i^{-1} k_i x + \sum h_i^* B_r l_i^{-1} u - \sum h_i^* B_r l_i^{-1} x - \sum h_i^* B_r l_i^{-1} u + \sum h_i B_r l_i^{-1}(-k_i x + l_i r)$$

$$= A_m e + \sum h_i^* B_r \left(\widetilde{l_i^{-1} k_i}\right) + \sum h_i^* B_r \left(l_i^{*-1} - l_i^{-1}\right) u + \sum \tilde{h}_i B_r l_i^{-1} k_i x + \sum h_i^* B_r l_i^{-1} u + \sum h_i B_r l_i^{-1}(-k_i x + l_i r)$$

$$= A_m e + \sum \tilde{h}_i B_r (\widetilde{l_i^{-1}} k_i) + \sum \tilde{h}_i B_r (l_i^{*-1} - l_i) u + \sum h_i B_r \left(\widetilde{l_i^{-1}} k_i\right) + \sum h_i B_r \left(l_i^{*-1} - l_i^{-1}\right) u + \sum \tilde{h}_i B_r l_i^{-1} k_i x + \sum \tilde{h}_i B_r l_i^{-1} u + \sum h_i B_r l_i^{-1} u + \sum h_i B_r l_i^{-1} l_i r$$

Obtain $\tilde{h}_i$ by using Taylor expansion for h_i^*:

$$h_i^* = h_i + \frac{\partial h_i}{\partial C_i} \Delta C + \frac{\partial h_i}{\partial \sigma_i} \Delta \sigma + H \cdot O \cdot T \quad (12)$$

$$\tilde{h}_i = h_i^* - h_i = \frac{\partial h_i}{\partial C_i} \Delta C + \frac{\partial h_i}{\partial \sigma_i} \Delta \sigma \quad (13)$$

$$\sum \tilde{h}_i . B_r . l_i^{-1} . k_i . x = \sum (h_i + \frac{\partial h_i}{\partial C} \,|_c \Delta C + \frac{\partial h_i}{\partial \sigma} \,|_c \Delta \sigma + H.O.T) B_r l_i^{-1} k_i x \quad (14)$$

And

$$\sum \tilde{h}_i . B_r . l_i^{-1} . u = \sum (h_i + \frac{\partial h_i}{\partial C^*} \,|_c \Delta C + \frac{\partial h_i}{\partial \sigma^*} \,|_c \Delta \sigma + H.O.T) B_r l_i^{-1} k_i x \quad (15)$$

According to the Lyapunov theory

$$V = e_m{}^T . P . e_m + \sum \tilde{k}_i{}^T . |l_i^*|^{-1} . \tilde{k}_i + \sum \tilde{l}_i{}^2 . |l_i^*|^{-1} + \sum \tilde{C}_i{}^T . \tilde{C}_i + \sum \tilde{\sigma}_i{}^T . \tilde{\sigma}_i \quad (16)$$

Adaptive laws for data center and variance reached by (16). See (17).

$$\dot{\tilde{C}}_i{}^T = \gamma . \frac{\partial h_i}{\partial C} . B_r . l_i^{-1} . u$$

$$\dot{\tilde{\sigma}}_i{}^T = \gamma . \frac{\partial h_i}{\partial \sigma} . B_r . l_i^{-1} . k_i . x \quad (17)$$

These equations give numerous membership functions, which would be stable and optimized.

4. Simulation of the Proposed Method on a Flexible Joint Robot and Chua's Circulate

This part simulates The proposed method on a flexible joint robot system. Fig.1 shows this system. After that, in part 5, this method simulated Chua's circulation.

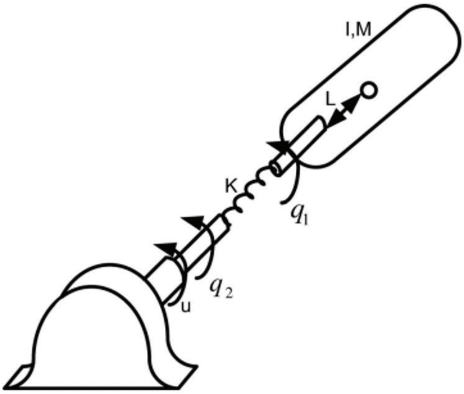


Fig. 1: Single Flexible Joint Machine
(Khanesar, M.A., Kaynak, O. and Teshnehlab, M., 2011)

Define $q = \{q_1 . \dot{q}_1 . q_2 . \dot{q}_2\}$ as the set of the generalized coordinates for the system. We have:

1. $q_2 = -(\frac{1}{m})\theta_1$is the angular displacement of the rotor, and m is the gear ratio.
2. q_1: is the angle of the link.
3. $q_1 - q_2$: is the elastic displacement of the link.

Write the equations of the system's model as (18):

$$I\ddot{q}_1 + Mgl\sin(q_1) + K(q_1 - q_2) = 0$$
$$J\ddot{q}_2 - K(q_1 - q_2) = u \qquad (19)$$

The state-space representation of this system is:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{Mgl}{I}\sin(x_1) - \frac{K}{I}(x_1 - x_3)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{K}{J}(x_1 - x_3) + \frac{1}{J}u \qquad (20)$$

It assumed that $x_1 = q_1$ and $x_3 = q_2$.

Because of feedback linearization, this coordination of the system changed.

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = z_3$$
$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -\left(\frac{MgL}{I}\cos z_1 + \frac{k}{I} + \frac{K}{J}\right) z_3 + \frac{MgL}{I}\left(z_2^2 - \frac{K}{J}\right)\sin z_1 + \frac{K}{IJ}u \qquad (21)$$

The numerical values of the parameters considered in the simulation studies are:

The fuzzy membership function reached (21) for modeling a flexible joint robot system. The T-S fuzzy model of the system is as follows:

Rule 1: IF z_1 IS ABOUT (C_{011}), AND z_2 IS ABOUT (C_{021})
THEN $\dot{z} = A_1 z + B_1 u$
Rule 2: IF z_1 IS ABOUT (C_{011}), AND z_2 IS ABOUT (C_{022})
THEN $\dot{z} = A_2 z + B_2 u$
Rule 3: IF z_1 IS ABOUT (C_{011}), AND z_2 IS ABOUT (C_{023})
THEN $\dot{z} = A_3 z + B_3 u$
Rule 4: IF z_1 IS ABOUT (C_{012}), AND z_2 IS ABOUT (C_{021})
THEN $\dot{z} = A_4 z + B_4 u$
Rule 5: IF z_1 IS ABOUT (C_{012}), AND z_2 IS ABOUT (C_{022})
THEN $\dot{z} = A_5 z + B_5 u$
Rule 6: IF z_1 IS ABOUT (C_{012}), AND z_2 IS ABOUT (C_{023})
THEN $\dot{z} = A_6 z + B_6 u$
Rule 7: IF z_1 IS ABOUT (C_{013}), AND z_2 IS ABOUT (C_{021})
THEN $\dot{z} = A_7 z + B_7 u$
Rule 8: IF z_1 IS ABOUT (C_{013}), AND z_2 IS ABOUT (C_{022})
THEN $\dot{z} = A_8 z + B_8 u$
Rule 9: IF z_1 IS ABOUT (C_{013}), AND z_2 IS ABOUT (C_{023})
THEN $\dot{z} = A_9 z + B_9 u$
Where:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9\cdot 8 & 0 & -11\cdot 8 & 0 \end{bmatrix}$$

$$A_2 = A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 86\cdot 92 & 0 & -11\cdot 8 & 0 \end{bmatrix}$$

$$A_4 = A_7 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 9\cdot 8 & 0 & 7\cdot 8 & 0 \end{bmatrix}$$

$$A_5 = A_6 = A_8 = A_9 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -86\cdot 92 & 0 & 7\cdot 8 & 0 \end{bmatrix}$$

$$B_i = [0 \quad 0 \quad 0 \quad 1]^T. i = 1.2. \dots .9$$

Suppose A_m In reference model (4) in this case:

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -160 & -192 & -82 & -15 \end{bmatrix} \quad (22)$$

e desired poles of the system are as $[-2.-5.-4.-4]$. So, taking Q as

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (23)$$

Then:

$$P = \begin{bmatrix} 0\cdot 9948 & -0\cdot 5000 & -0\cdot 9050 & 0\cdot 5000 \\ -0\cdot 5000 & 0\cdot 9050 & -0\cdot 5000 & -3\cdot 5511 \\ -0\cdot 9050 & -0\cdot 5000 & 3\cdot 5511 & -0\cdot 5000 \\ 0\cdot 5000 & -3\cdot 5511 & -0\cdot 5000 & 42\cdot 8880 \end{bmatrix} \quad (24)$$

The initial values of the gain of K_i and l_i are as follows:

$$B_i K_i = A_i - A_m \,.\, i = 1.2. \ldots .9 \quad (25)$$

$$B_i l_i = B_r \,.\, i = 1.2. \ldots .9 \quad (26)$$

Finally, Simulations are done by software for implementation. First, the reference model signal is a stop signal, and the aim is to examine regulation and tracking response. The adaptation gains are below.

Assume:

$$\gamma_1 = 0\cdot 001 \,.\, \gamma_2 = 0\cdot 1$$

$$\gamma_3 = 0\cdot 001 \,.\, \gamma_4 = 0\cdot 01$$

Figure 2 shows the regulation response of the controller when applied to the flexible joint robot.

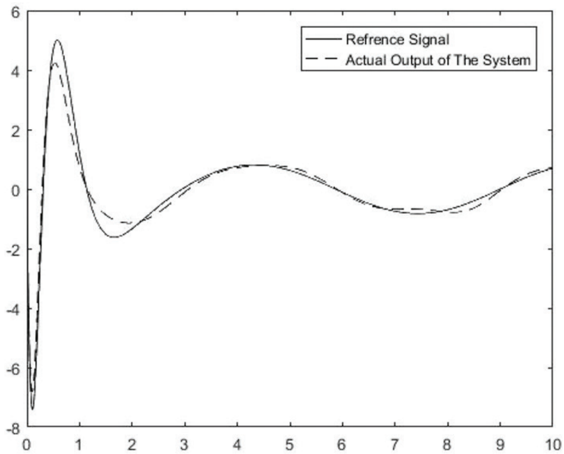


Fig. 2: Regulation response of the controller when applied to the flexible joint

After that, Figure 3 shows the tracking response of the controller. The reference signal is the step signal.

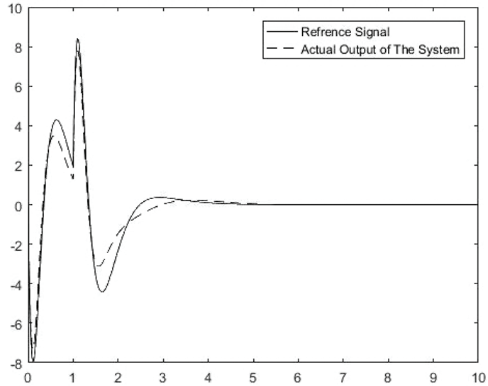
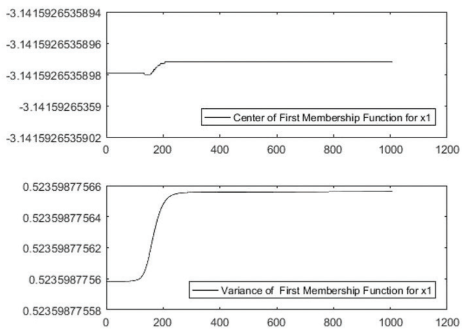


Fig. 3: Tracking response of the controller when applied to the flexible joint

The most critical issue in this article was the uncertainty in membership function. The point is to find optimal amounts of center and variance of membership function. Figures 4 and 5 show the optimal amounts of center and variance in the first membership function for x_1 andx_2. Figures 6 and 7 demonstrate the norm of center and variance. By comparing the results in figures with figures in (Khanesar, M.A. and Hosseini, M., 2017), we conclude that variable parameters could work.



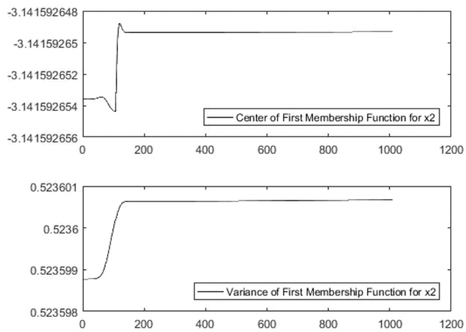


Fig. 4: The first membership function for x_1 *Fig. 5: The first membership function for* x_2

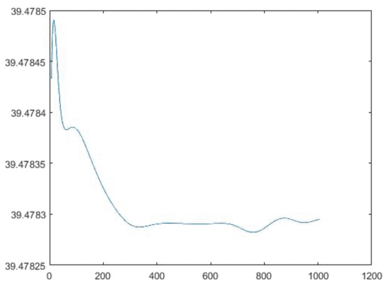


Fig. 6: The norm of the center of membership function for flexible joint

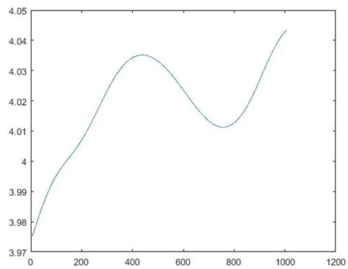


Fig. 7: The norm of variance of membership function for flexible joint

For raising assurance, this method applies to Chua's circulation. Results are improved.

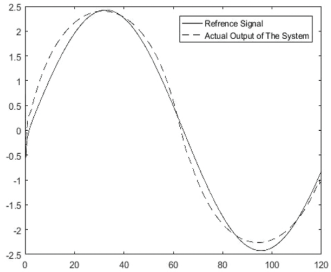


Fig. 8: Regulation response of the controller when applied to the Chua's Circulate

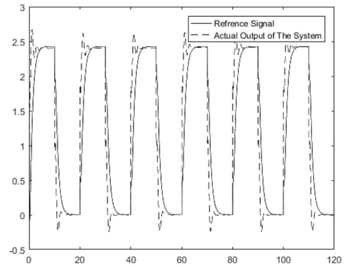


Fig. 9: Tracking step signal when applied to the Chua's Circulate

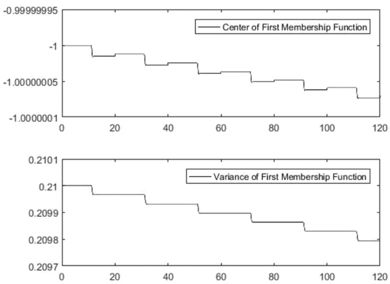


Fig. 10: The first membership function for x_1

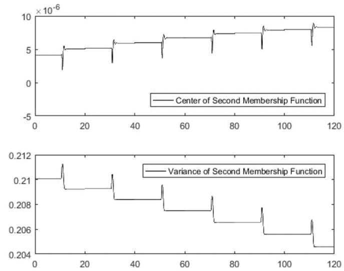


Fig. 11: The first membership function for x_2

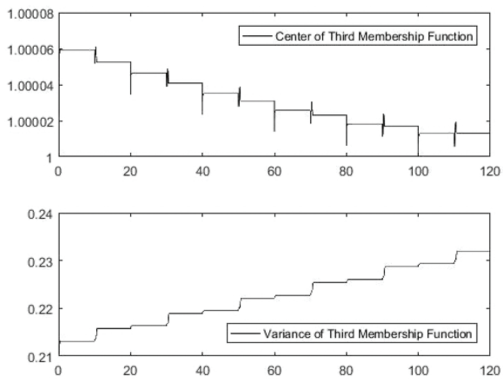


Fig. 12: The first membership function for x_3

Conclusion

Generally, adaptive controllers can be considered in two types. The first is direct, and the second is known as indirect controller. Using a direct model is necessary to know, Update the system's parameters directly. In indirect model parameters, though, regulation is made after estimating the controller parameters. We used a direct model. Because of the Lyapunov function and its stability, we can write differential equations and, by using these equations, want to find optimal values for our parameters. Almost in all articles, the membership function is similar. Like the Gaussian model and the values of variance and center are constant. At first, finding the optimal values of these parameters and while simulating the problem, these parameters are constant. However, because of the adaptive controller, the controller's parameters are updated and, each time, have different values.

For this reason, the possibility exists constant values of the membership function cannot be helpful. So need to update these values too. At the same time, for using the fuzzy identifiers and adaptation laws to update the controller coefficients, amounts of center and variance of membership function are updated too and get the optimal values every time. So The controller can provide the best response to the reference inputs, as shown in Figures 2 and 3.

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